DFG

CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD
Faculty of Physics

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U(1) lattice gauge theory on thimble tangent spaces

Felix Ziegler, CP3-Origins

Joint work with J. M. Pawlowski, M. Scherzer, C. Schmidt and F. Ziesché

SIGN 2019, Odense, 2. - 6. September 2019

Overview

- U(1) lattice gauge theory with a sign problem
- Motivation for local update algorithms
- Thermodynamic limit
- Simulating on tangents
- One-plaquette model and analytic solution
- Numerical results
- Perspectives

U(1) lattice gauge theory at complex coupling

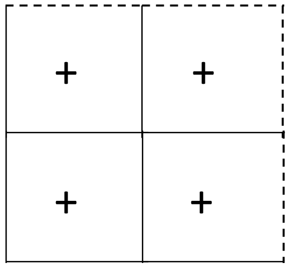
$$Z = \prod_{k=1}^{2V} \int_{U(1)} dU_k \; e^{-S[U]}$$

$$\prod_{k=1}^{V} P_k = 1$$

$$S[U] = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{2} \left(P_{x,\mu\nu}[U] + P_{x,\mu\nu}[U]^{-1} \right) \right)$$

$$P_{x,\mu\nu}[U] = U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}{}^{-1} U_{x,\nu}{}^{-1}$$

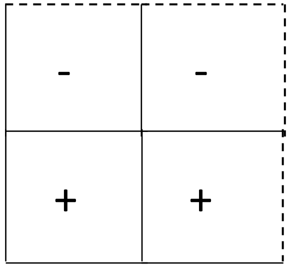
Thimbles and critical manifolds (see talk bei F. Ziesché)



$S_R = 0$

$P_k = \pm 1$

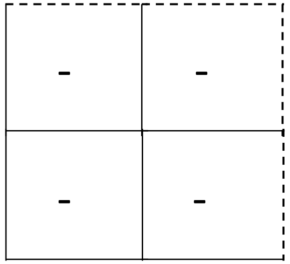
$S_R = 4k\beta_R, k = 0, \ldots, V/2$



(x6)

$S_R = 4\beta_R$

Hierarchy of thimbles



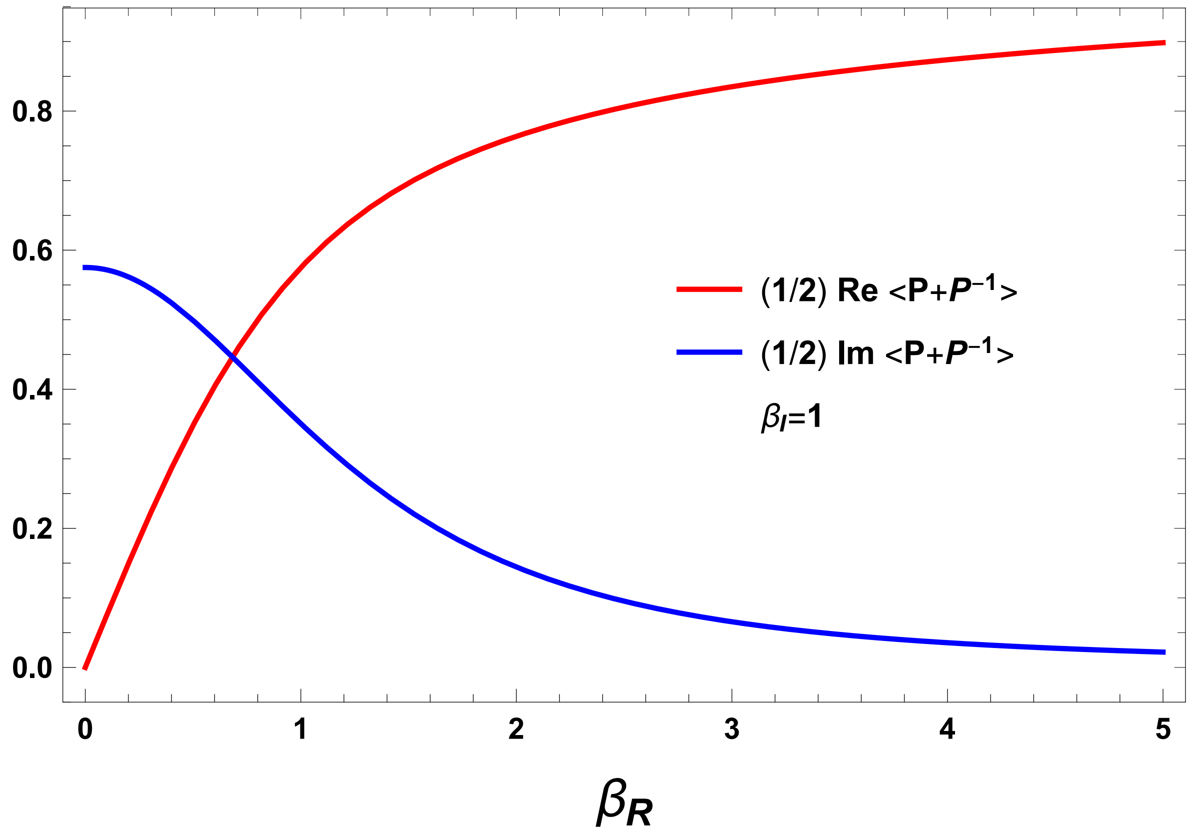
$S_R = 8\beta_R$

Thermodynamic limit

$$Z \xrightarrow{V\to\infty} \left(\int_{U(1)} dP\, e^{\beta/2(P+P^{-1})}\right)^V = I_0\,(\beta)^V$$

$$\left(\frac{1}{2}\right) < P + P^{-1} > = \frac{I_1(\beta)}{I_0(\beta)}$$

See Rothe‘s textbook



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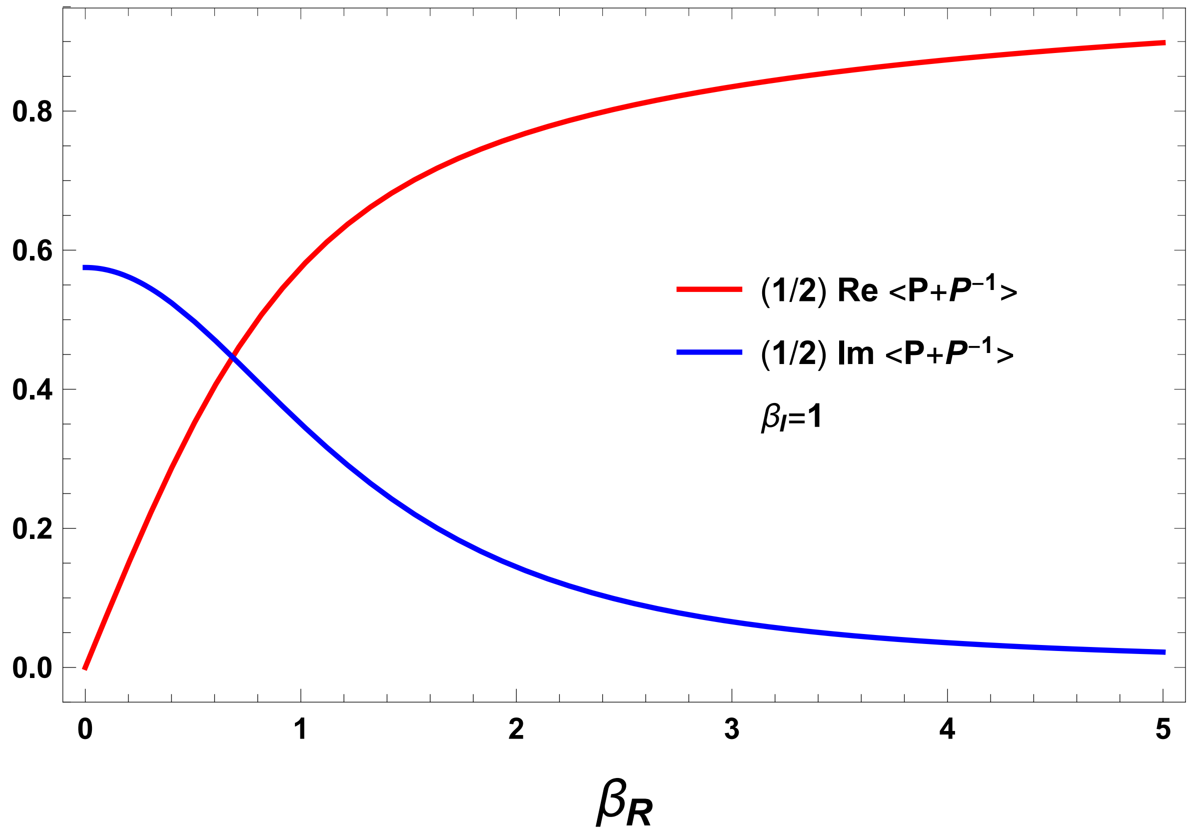
one-plaquette model

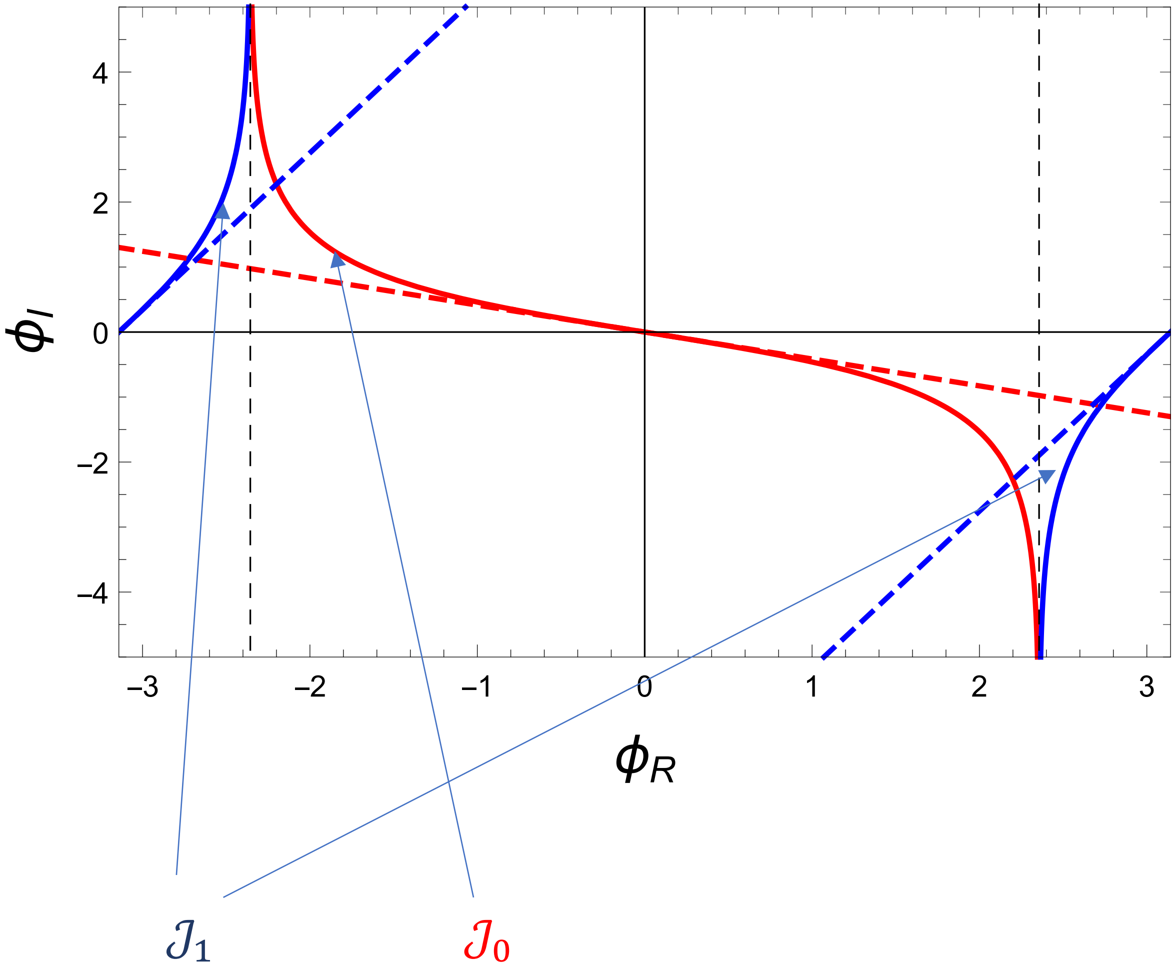


Thermodynamic limit and Thimbles

$$Z \xrightarrow{V\to\infty} \left(\int_{U(1)} dP\, e^{\beta/2(P+P^{-1})} \right)^V = I_0\,(\beta)^V$$

$$= \left(\int_{\mathcal{J}_0} dP\, e^{\beta/2(P+P^{-1})} + \int_{\mathcal{J}_1} dP\, e^{\beta/2(P+P^{-1})} \right)$$

$$=: (Z_0 + Z_1)^V$$



Thermodynamic limit and tangent space

$$Z \xrightarrow{V\to\infty} \left(\int_{U(1)} dP\, e^{\beta/2(P+P^{-1})}\right)^V = I_0\,(\beta)^V$$

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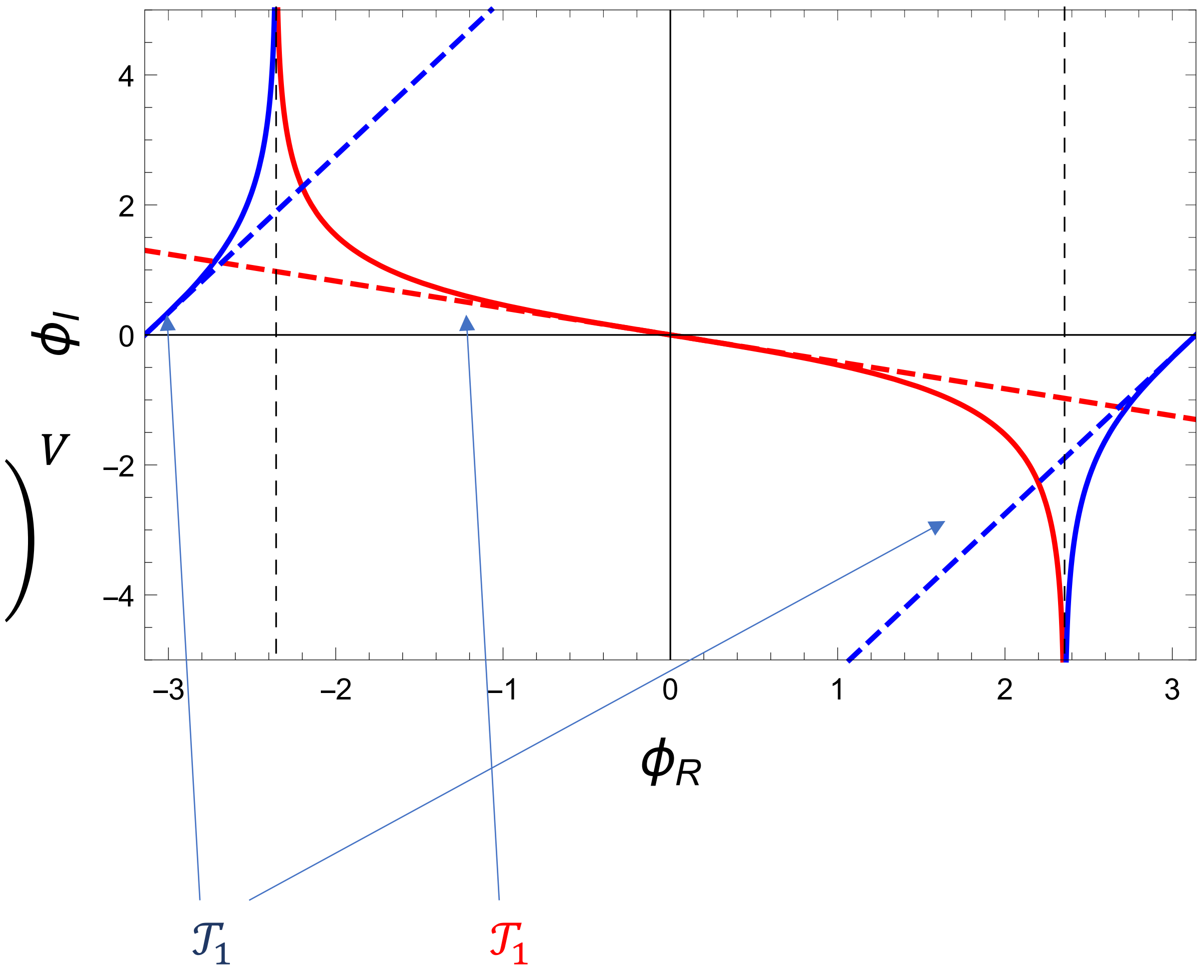
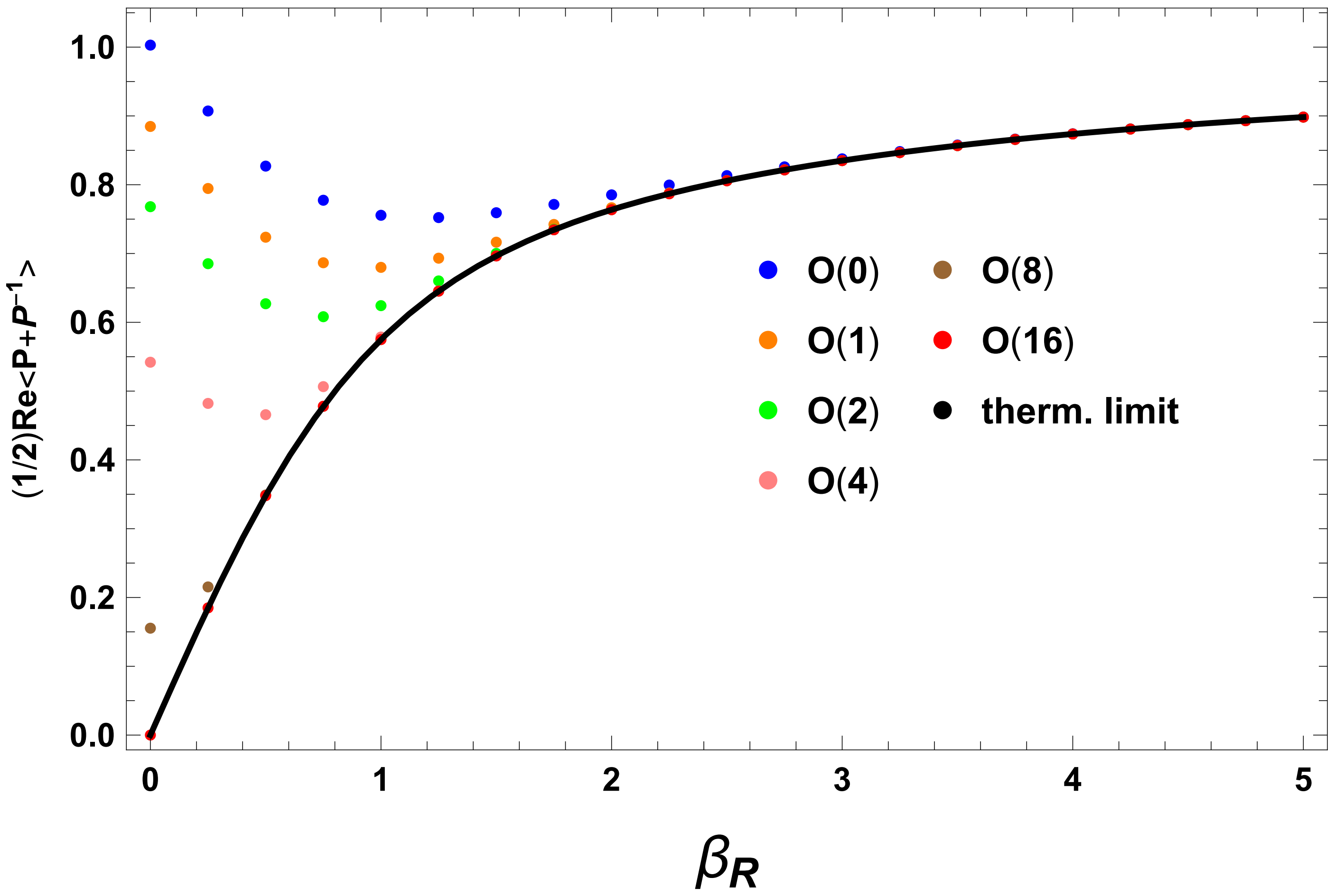


Thermodynamic limit and tangent space



Idea for local tangent MC algorithm

- product manifold of tangents homotopy-equivalent to $U(1)^{2V}$
- Recipe: update single link such that surrounding plaquettes lie on the tangent spaces
- Exploit hierarchy of thimbles -> see talk by F. Ziesché
- Prospect: numerically cheap
- Challenge: Thimble structure intricate

Idea for local tangent MC algorithm

- product manifold of tangents homotopy-equivalent to $U(1)^{2V}$

- Recipe: update single link such that surrounding plaquettes lie on the tangent spaces

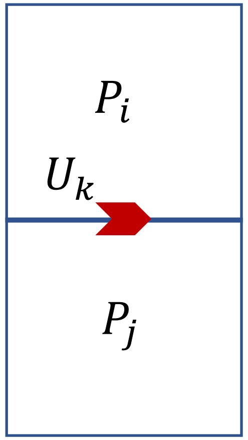
Important for this procedure: main tangent carries the largest weight!

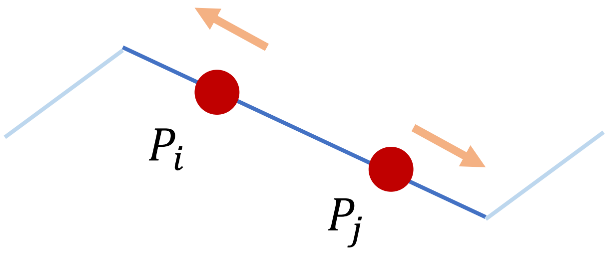
$$Z = Z_{\mathcal{J}_0} + Z_{\mathcal{J}_1} + Z_{\mathcal{J}_2} + \ldots + Z_{\mathcal{J}_N}$$

Mind relative weights!!!

Update on the main tangent

1. Consider tangent space for single plaquette
2. Pick a link U_k
3. Select a random angle $\Delta\phi_k$
4. Set U_k to $U_k{}' = \exp(i(\Delta\phi_k + \phi_k))$
5. Metropolis acc.-reject step. Reject if plaquettes are driven out of the main tangent.





Proposal for Metropolis acc.-rej. step

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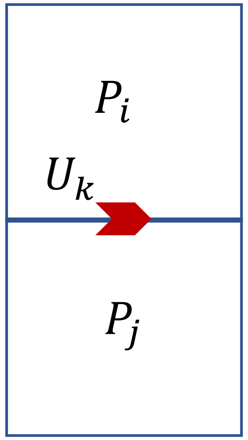
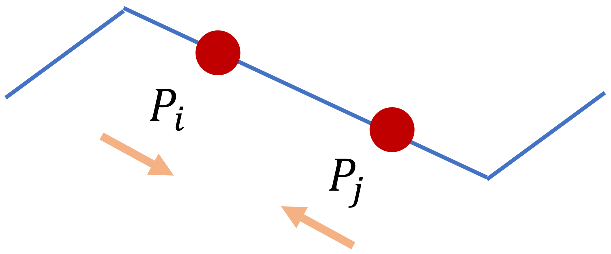




Proposal for Metropolis acc.-rej. step

Relative weights

- Account for relative weights Z_1/Z_0 between main and sub-leading tangent using reweighting technique for thimbles

  Bluecher, Pawlowski, Scherzer, Stamatescu, FZ, SciPost Phys. 5, (2018)

- Given a mapping $f: \mathcal{T}_0 \longrightarrow \mathcal{T}_1$

$$\frac{Z_1}{Z_0} = \frac{\int_{\mathcal{T}_0} dU\, e^{-S[f(U)]+S[U]}\, \det(f)\, e^{-S[U]}}{\int_{\mathcal{T}_0} dU\, e^{-S[U]}}$$

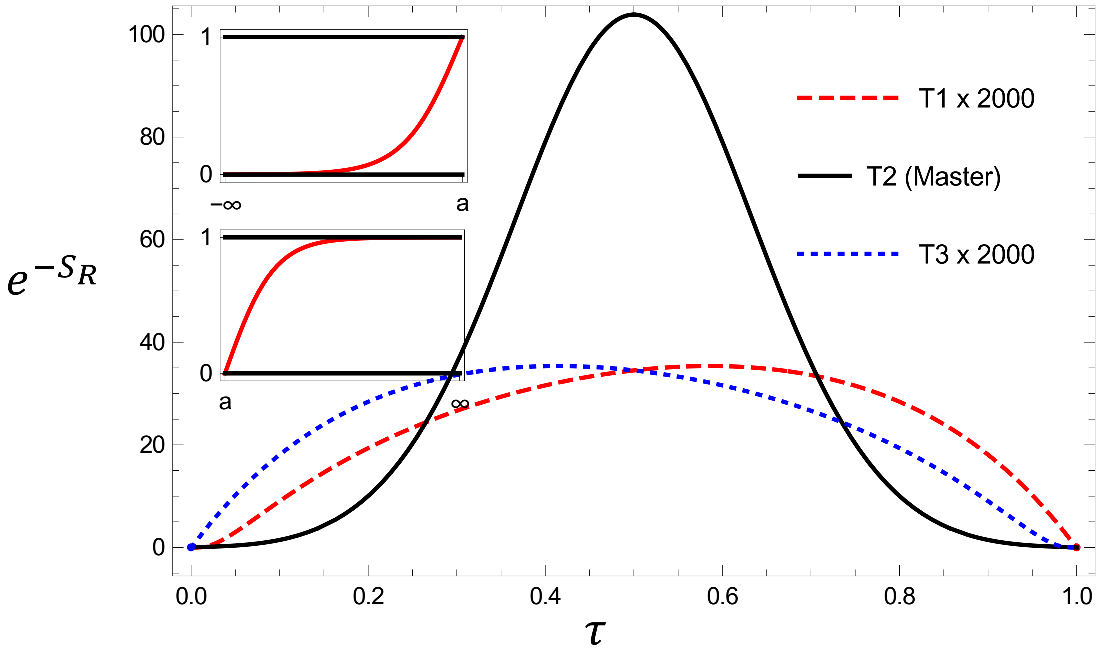
$$= \left\langle e^{-S[f(U)]+S[U]} \det(f) \right\rangle_0$$

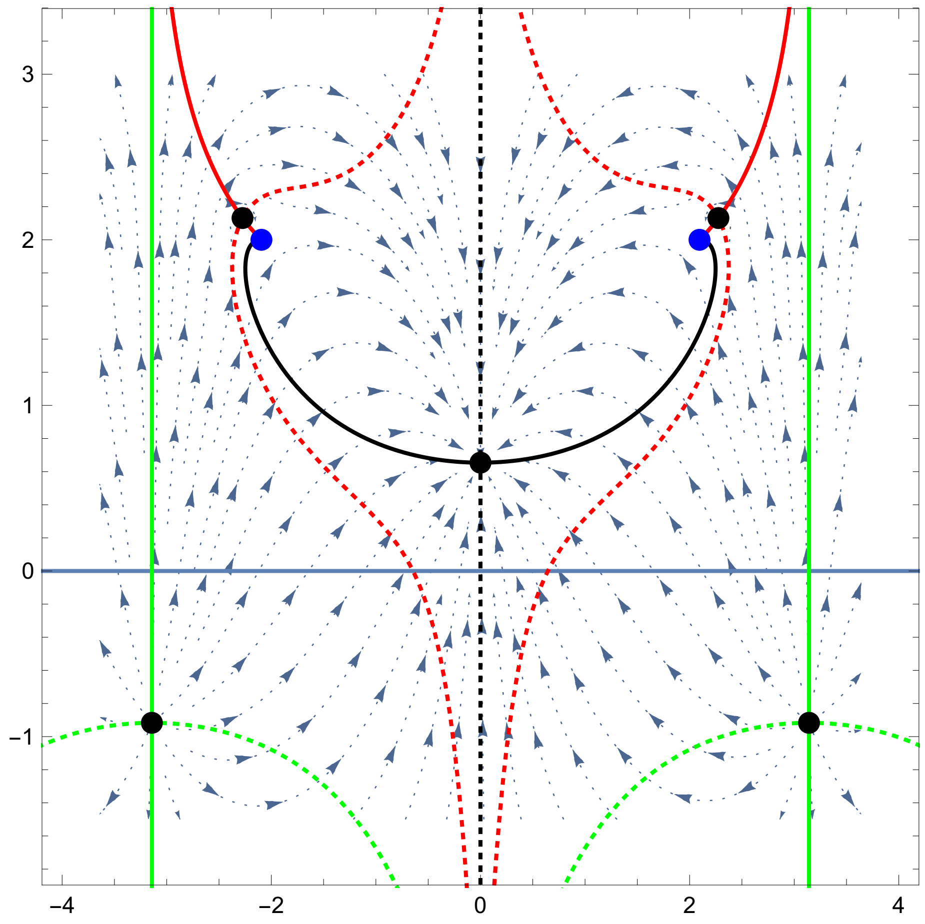
- Challenge: find f
- Good news: tangent space linear ⇒ Jacobian constant

Reweighting

Use reweighting technique for thimbles

Bluecher, Pawlowski, Scherzer, Stamatescu, FZ, SciPost Phys. 5, (2018)





U(1) one-link model with finite μ,
see Aarts, Stamatescu, JHEP 0809 (2008)

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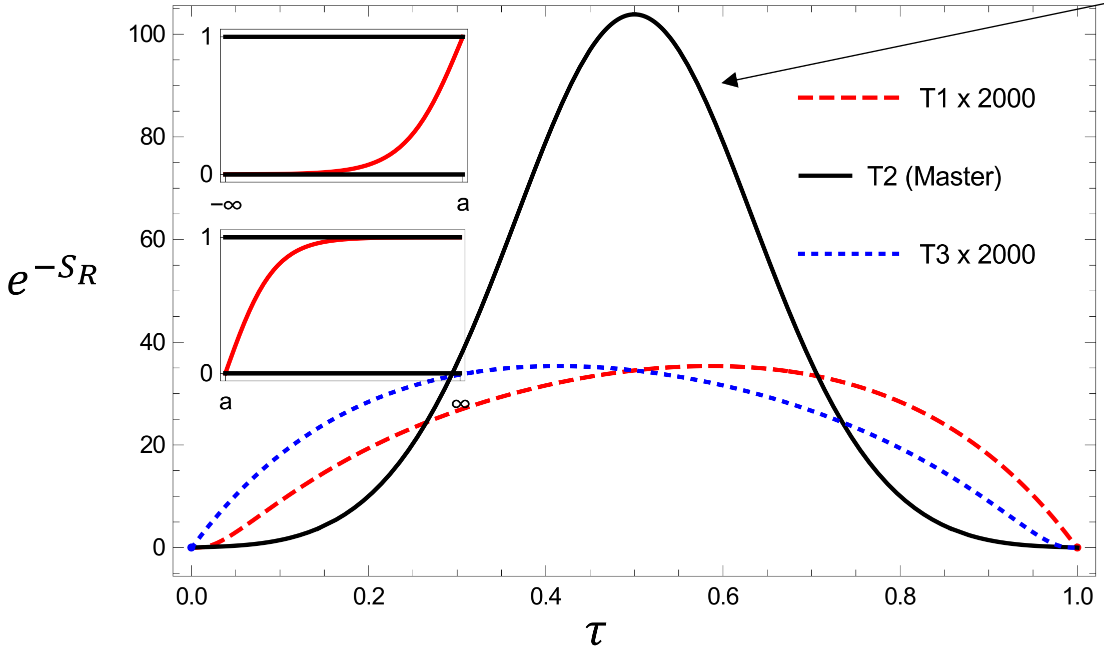

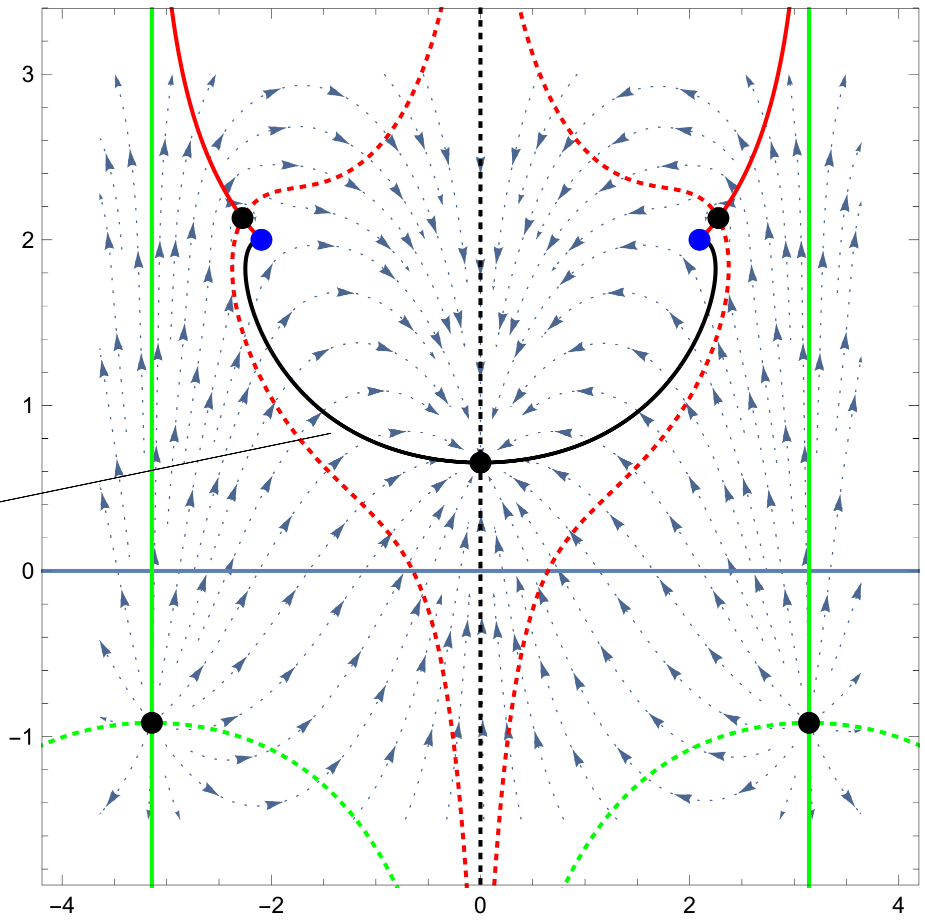

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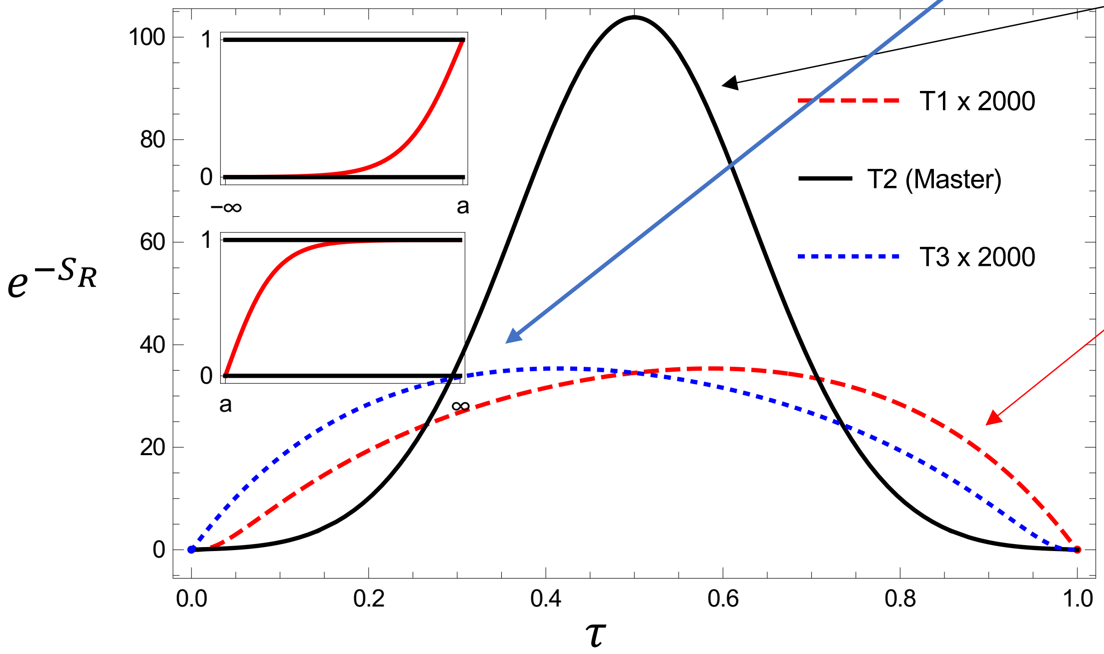

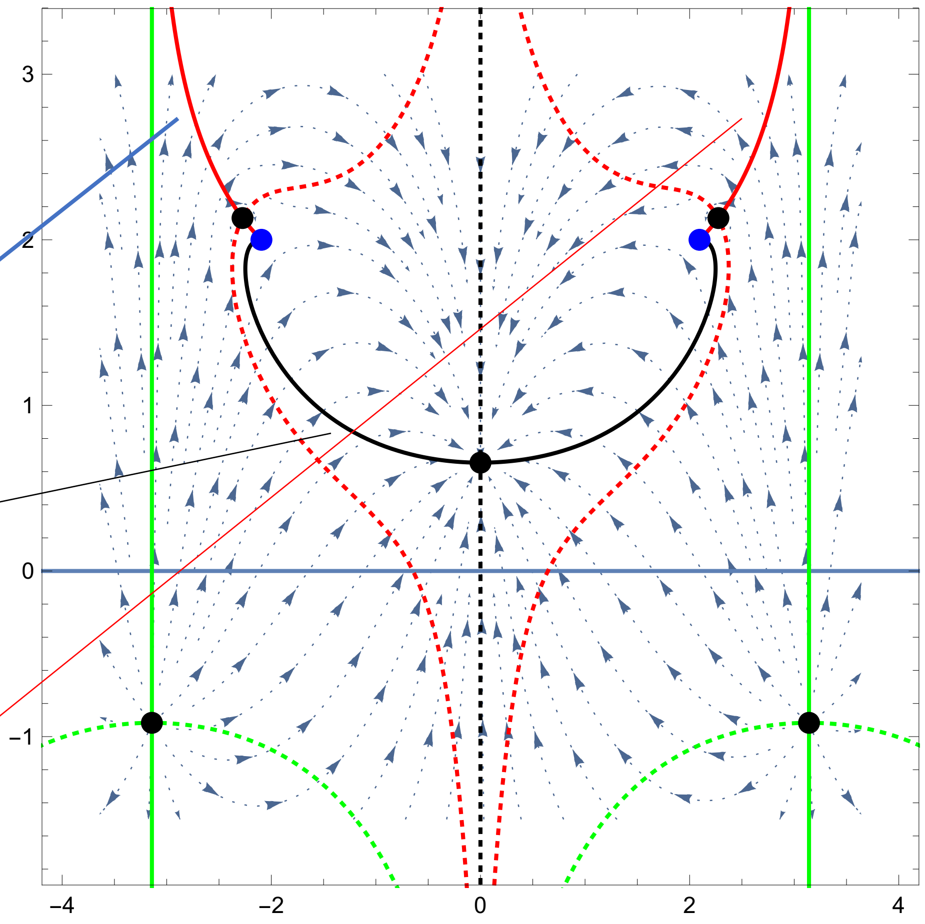

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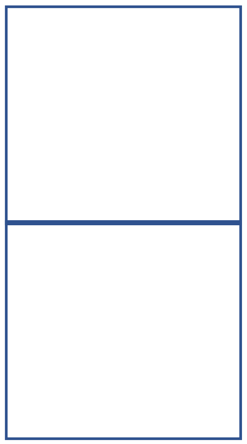
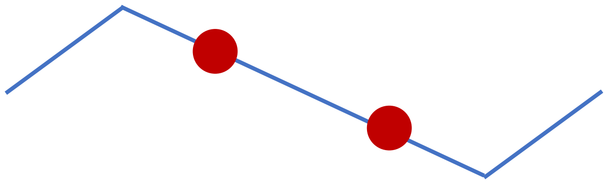
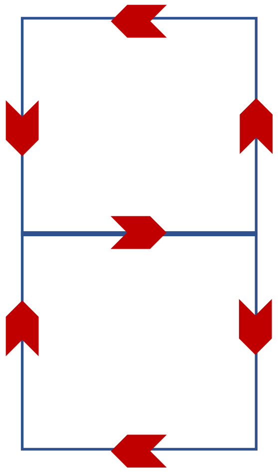
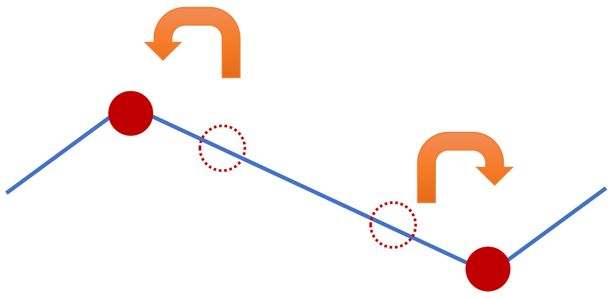
How to sample the sub-leading tangents





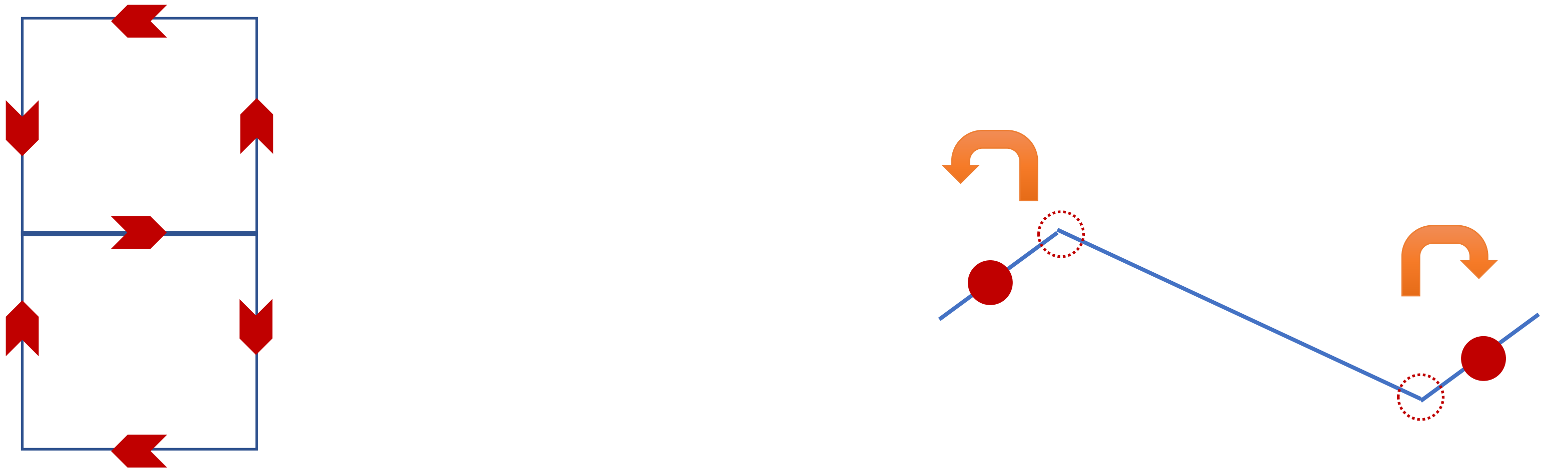
How to sample the sub-leading tangents





Offset change in plaquette in the surrounding links.

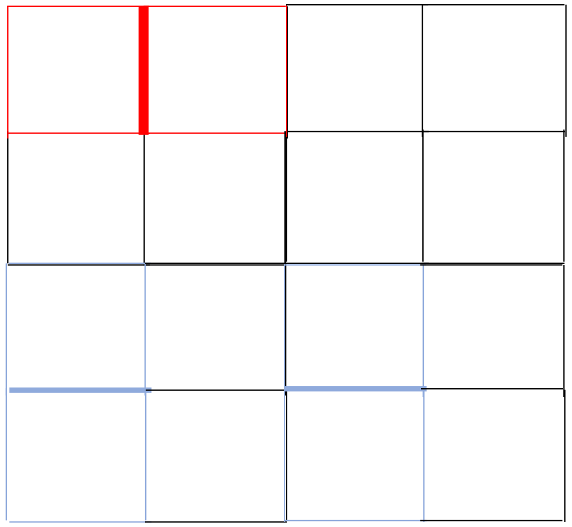
How to sample the sub-leading tangents



Offset change in plaquette in the surrounding links.

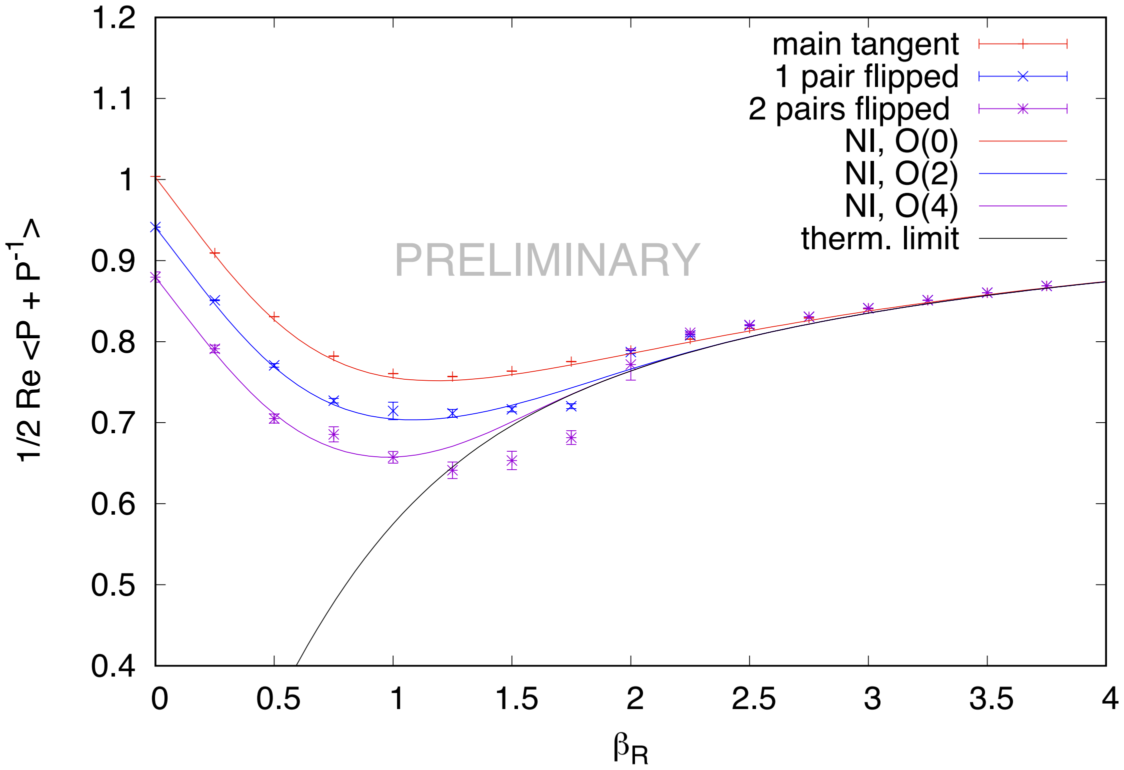
Current version of the algorithm

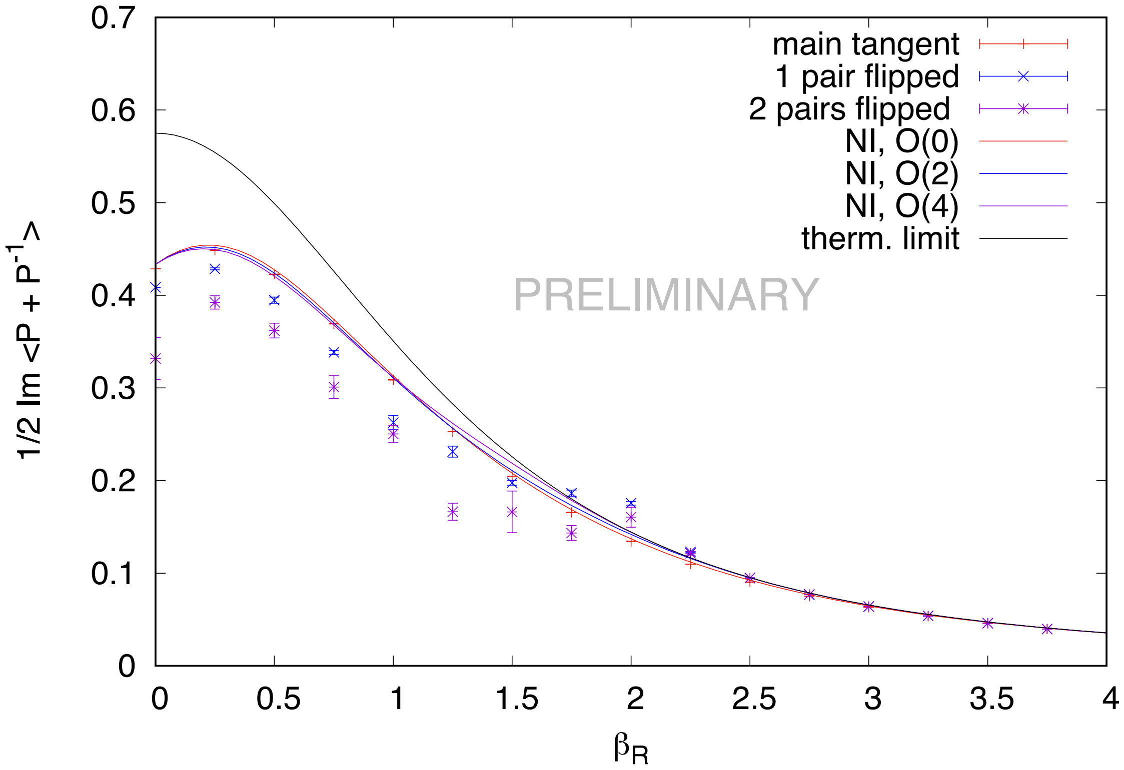
- LO: main tangent
- NLO: single flipped plaq. pair
- NNLO: flip 2 disjoint plaq. pairs



<u>Degeneracy factors</u> for the configurations on the sub-leading tangents!

Test case - 8x8 lattice for fixed $\beta_I = 1$





Diagnostics

- links: see blackboard

- Unitarity norm rises monotonously -> gauge cooling

  inspired by Seiler, Sexty, Stamatescu, Phys.Lett. B723 (2013)

- Deviation at small β_R: missing configurations

Perspectives

- Include more tangent configurations
- Extension to SU(N) gauge groups
- Fermions and finite density
- Real-time physics on the Schwinger-Keldysh contour

Thank you very much!